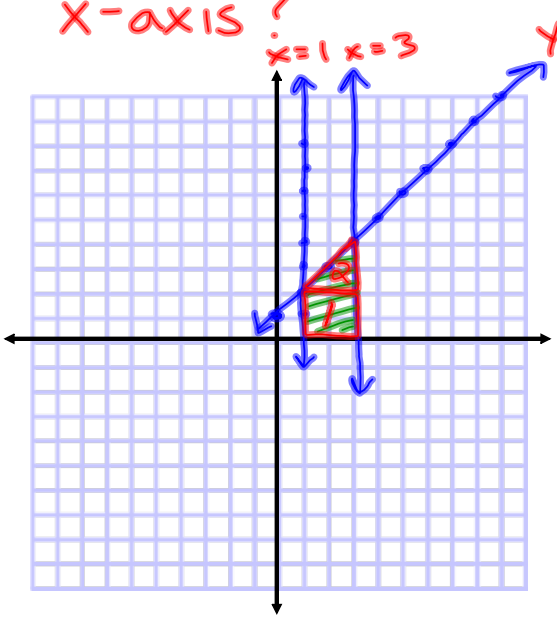
4/19/17

AIM: How do we find the area under a curve?

1) Graph $y = x+1$, $x = 1$, $x = 3$

What is the area contained under the graph and above the x-axis?



Area 1 = $2 \times 2 = 4$

Area 2 = $\frac{2 \times 2}{2} = 2$

Area 1+2 = 6 units^2

2) Evaluate:

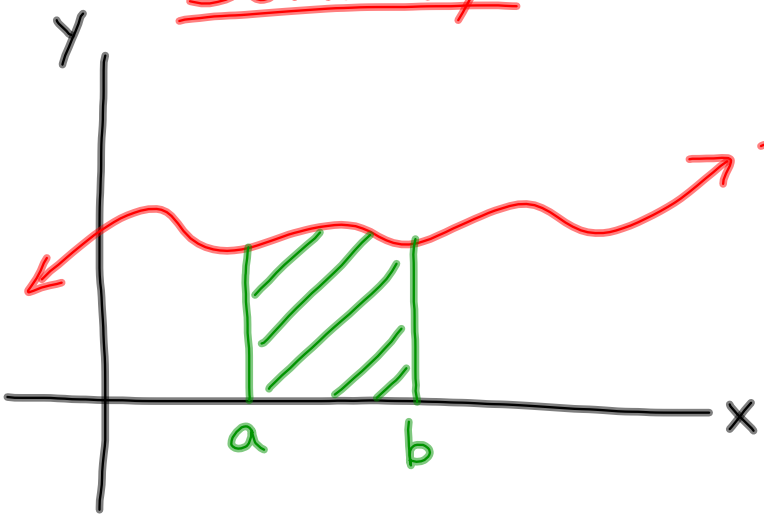
$$\int_1^3 (x+1)\,dx = \frac{x^2}{2} + x + c \Big]_1^3$$

$$\left(\frac{3^2}{2} + 3 + c\right) - \left(\frac{1^2}{2} + 1 + c\right)$$

$$\left(\frac{9}{2} + 3\right) - \left(\frac{1}{2} + 1\right)$$

$$\left(\frac{15}{2}\right) - \left(\frac{3}{2}\right) = \frac{12}{2} = 6$$

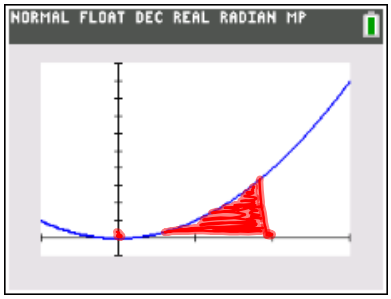
Summary:



Area between f(x) and the x-axis between [a, b] is:

$$\int_a^b f(x)\,dx$$

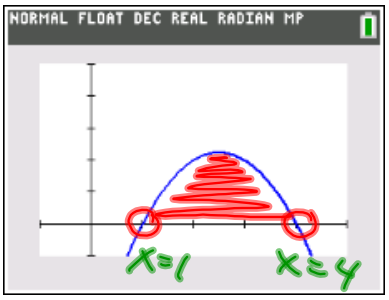
3) What is the area between $f(x) = x^2$ and the x-axis on the interval $[0,2]$?



$$\text{Area} = \int_0^2 x^2 \, dx$$

$$\text{Area} = \frac{x^3}{3}\Big]_0^2 = \frac{2^3}{3} - \frac{0^3}{3} = \boxed{\frac{8}{3} \text{ units}^2}$$

4) What is the area in the 1st quadrant between $y = -x^2+5x-4$ and the x-axis?



Need the zeroes

$-x^2+5x-4=0$

$x^2-5x+4=0$

$(x-1)(x-4)=0$

$x=1 \quad x=4$

$$\text{Area} = \int_1^4 (-x^2+5x-4)\,dx$$

$$\text{Area} = -\frac{x^3}{3}+\frac{5x^2}{2}-4x\Big]_1^4$$

$$\left(-\frac{4^3}{3}+\frac{5(4)^2}{2}-4(4)\right)-\left(\frac{-1^3}{3}+\frac{5(1)^2}{2}-4(1)\right)$$

$$\left(-\frac{64}{3}+\frac{80}{2}-16\right)-\left(-\frac{1}{3}+\frac{5}{2}-4\right)$$

$$= \boxed{4.5 \text{ units}^2}$$